singular value decomposition (SVD)

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Abstract

The singular value decomposition (SVD) is a factorization of a matrix $\mathbf{A} \in \mathbb{R}^{m \times d}$ of the form $\mathbf{A} = \mathbf{V}\mathbf{\Lambda}\mathbf{U}^\top$ with orthonormal matrices $\mathbf{V}$ and $\mathbf{U}$. The matrix $\mathbf{\Lambda}$ is nonzero only along its main diagonal. The diagonal entries of $\mathbf{\Lambda}$ are nonnegative and referred to as singular values. In contrast to an eigenvalue decomposition (EVD), which requires a square diagonalizable matrix, an SVD exists for every matrix.

Definition

The SVD for a matrix $\mathbf{A} \in \mathbb{R}^{m \times d}$ is a factorization of the following form:

$$\mathbf{A} = \mathbf{V}\mathbf{\Lambda}\mathbf{U}^\top$$

with orthonormal matrices

$$\mathbf{V} = \left(\mathbf{v}^{(1)}, \ldots, \mathbf{v}^{(m)}\right) \in \mathbb{R}^{m \times m}, \qquad \mathbf{U} = \left(\mathbf{u}^{(1)}, \ldots, \mathbf{u}^{(d)}\right) \in \mathbb{R}^{d \times d}$$

(Golub and Van Loan, 2013) (see Fig. 1).

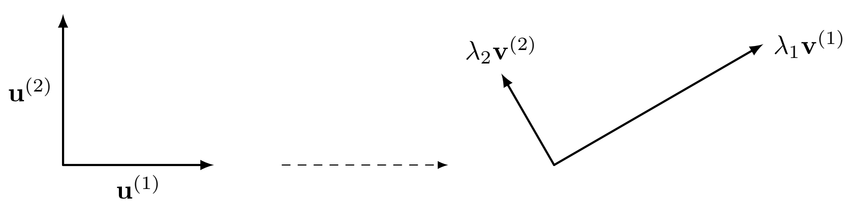

Figure 1: Orthonormal matrices $\mathbf{V}$ and $\mathbf{U}$

The matrix $\mathbf{\Lambda} \in \mathbb{R}^{m \times d}$ is only nonzero along the main diagonal, whose entries $\Lambda_{j,j}$ are nonnegative and referred to as singular values.

Cross References

matrix

References

1. Golub and Van Loan (2013). Matrix Computations. 4th ed., The Johns Hopkins Univ. Press, Baltimore, MD, USA.