

random variable (RV)

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Abstract

A random variable (RV) is a function that maps each outcome of a random experiment to an element of a measurable space. Important types of RVs include binary RVs, discrete random variables (discrete RVs), real-valued RVs, random vectors, and random matrices. In machine learning (ML), data points are often interpreted as realizations of independent and identically distributed (i.i.d.) RVs, whose probability distribution then governs their typical properties.

Definition

An RV is a function that maps the outcomes of a random experiment to elements of a measurable space (Billingsley, 1986; Gray, 2009). Mathematically, an RV is a function $x : \Omega \rightarrow \mathcal{X}$ whose domain is the sample space Ω of a probability space and whose co-domain is a measurable space $\mathcal{X}$. Different types of RVs include

- binary RVs, which map each outcome to an element of a binary set (e.g., $\{-1, 1\}$ or $\{\text{cat}, \text{no cat}\}$);
- discrete random variables (discrete RVs), which take on values in a countable set (which can be finite or countably infinite);
- real-valued RVs, which take on values in the real numbers $\mathbb{R}$;
- random vectors, which map outcomes to the Euclidean space $\mathbb{R}^d$;
- random matrices, which map outcomes to a space of matrices $\mathbb{R}^{m \times d}$.

Probability theory uses the concept of measurable spaces to rigorously define and study the properties of collections of RVs (Billingsley, 1986).

Cross References

function, random experiment, sample space, probability space, vector, Euclidean space, probability, measurable, random vector, random matrix, sub-Gaussian RV

References

1. Billingsley (1986). Probability and Measure. 2nd ed., Wiley, New York, NY, USA.
2. Gray (2009). Probability, Random Processes, and Ergodic Properties. 2nd ed., Springer Science+Business Media, New York, NY, USA.